FULL MECHANICS SOLUTION

SINGLE CHOICE QUESTIONS

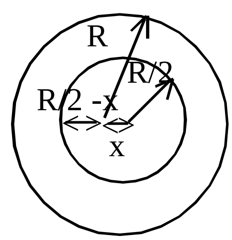
1. Minimum force to move the body $= \dfrac{\mu mg}{\sqrt{1+\mu^2}}$; $\Rightarrow \mu = \dfrac{3}{4}$

2. $\tau = mg(v\cos\theta t)$; $\Rightarrow \dfrac{dL}{dt} = mgv\cos\theta t$; $\Rightarrow L = \dfrac{mgv\cos\theta t^2}{2}$

3. $v^2 = \omega^2(A^2 - x^2)$ & $a = -\omega^2 x \Rightarrow v^2 = -\dfrac{a}{x}(A^2 - x^2)$

$\Rightarrow A = \sqrt{x^2 - \dfrac{xv^2}{a}}$.

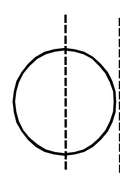
4. $-\left(\dfrac{R}{2} - x\right) = \dfrac{\frac{4}{3}\pi R^3(0) - \frac{4}{3}\pi\frac{R^3}{8}x}{\frac{4}{3}\pi\left(R^3 - \frac{R^3}{8}\right)}$; $\Rightarrow -\left(\dfrac{R}{2} - x\right) = -\dfrac{x}{7} \Rightarrow x = \dfrac{7R}{16}$



5. Using perpendicular axis theoram,

$2I_1 = M'R^2$; $I_1 = M'R^2/2$; $I_2 = \dfrac{M'R^2}{2} + M'R^2 = \dfrac{3}{2}M'R^2$

$I = \dfrac{I_2}{2} = \dfrac{3}{2}\dfrac{M'R^2}{2} = \dfrac{3}{2}MR^2$



6. Upward force due to tension on 3 kg block is F/2 which should be greater than 3g and upward force on 2 kg block is F/4 which should be grater than 2g

7. Avg. speed $3 = \dfrac{\sqrt{(v_r t)^2 + (v_{mr} t)^2}}{t}$

$\Rightarrow v_r^2 + 5 = 9$; $\Rightarrow v_r = 2m/s$

8. $v_{avg} = \dfrac{\frac{1}{2}x\frac{t}{2}xv + \frac{1}{2}xv}{t} = \dfrac{3v}{4}$

9. $\frac{m}{2}g - T = \frac{m}{2}a$ ... (i)

$T\cos 60^0 = \frac{ma}{\cos 60^0}$;.... (ii) Solving (i) and (ii) accelerating o ring $\frac{2g}{9}$

10. Work done by all the forces on the block equal to change in kinetic energy.

11. No effect of 'a' and 'g' on time period of spring pendulum.

12. Work done by friction = $\int \vec{F}.\vec{ds} = \int_0^x \mu mg\cos\theta \frac{dx}{\cos\theta}$

$= \mu mgx = 20J$

13. Conservation of energy $\frac{1}{2}mv^2 - mg\frac{\ell}{2} = mg\frac{\ell}{2} + m\sqrt{3}\left(2\ell\frac{\sqrt{3}}{2}\right)g$; $v = \sqrt{8g\ell}$

14. Conservation of momentum

$M\sqrt{2gh} - m\sqrt{2gh} = MV_1 + mV_2$ (1)

$-1 = \frac{V_1 - V_2}{2\sqrt{2gh}}$ .. (2)

$V_2 = 3\sqrt{2gh}$

$h' = \frac{V_2^2}{2g} = 9h$

15. FBD of 'B' and 'C'

$\Rightarrow T - 2g = 2a$ .... (1)

and 3g-T = 3a .....(2)

$T = \frac{12g}{5}$

For A $\Rightarrow T = mg$

$2x\frac{12g}{5} = mg \Rightarrow m = 4.8kg$

16. $W = \int_0^{\pi/2} f.Rd\theta = \frac{\mu mgR\pi}{2} = 1joule$

17. Let us assume cylinder is no moving then

$T + f_s = mg\sin\theta$

$T.R - f_s R = 0$

$$\Rightarrow f_s = \frac{mg\sqrt{3}}{4}$$

but $(f_s)_{max} = \mu N = \mu mg\cos\theta$

$= 0.4 \,.\, mg \text{ x } \frac{1}{2} = \frac{mg}{5}$

$\therefore (f_s) < (f_s)_{max}$, our assumption is wrong. So, friction existing must be kinetic

$f_k = \mu mg\cos\theta = 0.4 x mg x \frac{1}{2} = \frac{mg}{5}$

18. For full square about an axis pasing through 'O' = $\frac{M\ell^2}{6}$

by symmetry for remaining portion it must be $\frac{3}{4}\left(\frac{M\ell^2}{6}\right) = \frac{M\ell^2}{8}$ 29.B

19. Apply work-energy theorem

$mgz - \left\{\frac{1}{2}K\left(\frac{mg}{K}+z\right)^2 - \frac{1}{2}K\left(\frac{mg}{K}\right)^2\right\} + Fz = 0$; $\Rightarrow z = 2F/K$

20. $a = 2 + |t-2|$; for $t \le 2$; a = 2 - t + 2; a = 4-t

dv = (4-t)dt ; $v = 4t - t^2/2$ at t = 2, v = 6 m/s.

for t > 2 ; a = 2+ t -2 = t ; $\int_6^v dv = \int_2^t t\,dv$; v - 6= $\left[t^2/2\right]_2^t$

$v = \frac{t^2}{2} + 4$ at t = 4, v = 12 m/s.

MULTIPLE CHOICE QUESTIONS

21. If at any instant elongation in spring is x and velocity of block is v from equilibrium position then

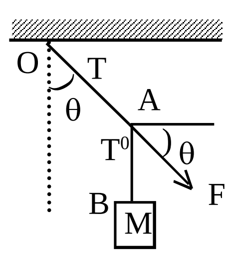
$$E_T \Rightarrow \frac{1}{2}kx^2 + \frac{1}{2}Mv^2 + \frac{1}{2}\int_0^L \frac{m}{L}dy\left(\frac{v}{L}y\right)^2$$

$= \frac{1}{2}kx^2 + \frac{1}{2}Mv^2 + \frac{1}{2}\frac{m}{L}\frac{v^2}{L}\frac{L^3}{3}$; $E_T \Rightarrow \frac{1}{2}kx^2 + \frac{1}{2}\left(M + \frac{m}{3}\right)v^2$

22. $T\sin\theta = F\cos\theta$

$T\cos\theta = F\sin\theta + T'$

$T' - F\sin\theta = Mg$



23. $$mg - \frac{2kq^2}{R^2} + T = \frac{mv^2}{R} \quad \text{... (i)}$$

$$-mgR + \frac{1}{2}mu^2 = mgR + \frac{1}{2}mv^2 \quad \text{.... (ii)}$$

$$\text{if } T = 0 \Rightarrow u_{min} = \sqrt{5gR - \frac{2Kq^2}{mR}}$$

24. $$F = -\frac{dU}{dx} = -5(2x - 4)$$

At mean position F = 0 $\Rightarrow x = 2m$

U_{min} = -20 J; a = -50 x 2 (x-2)

$\omega = 10 rad/sec$

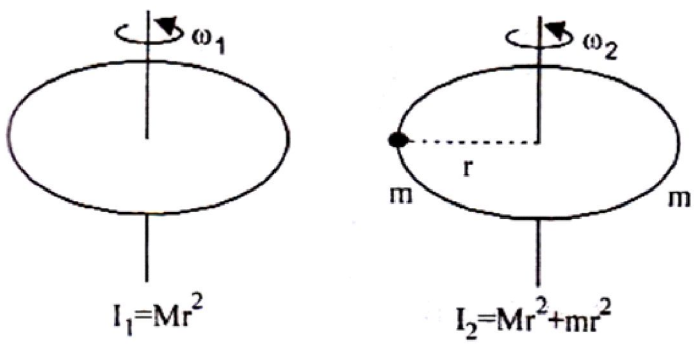
$T = \pi/5 sec$

25. If the volume immersed initially is (V/3). Then $\frac{V}{3}\rho g = mg$ .... (i)

If the volume immersed when the system accelerates is V' then

$$V'\rho\left(g + \frac{g}{2}\right) - mg = \frac{mg}{2} \Rightarrow V' = \frac{v}{3}$$

26. Since the objects are placed gently, therefore no external torque is acting on the system. Therefore angular momentum is constant.



i.e., $I_1 \omega_1 = I_2 \omega_2$

$$Mr^2 \times \omega_1 = (Mr^2 + 2mr^2)\omega_2$$

$$\therefore \omega_2 = \frac{M\omega}{M + 2m}$$

27. (a, b, c)

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Given that

$$\vec{\tau} = \vec{A} \times \vec{L} \Rightarrow \frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$$

From cross-product rule, $\frac{d\vec{L}}{dt}$ is always perpendicular to the plane containing $\vec{A}$ and $\vec{L}$.

By the dot product definition

$$\vec{L}.\vec{L} = L^2$$

Differentiating with respect to time

$$\vec{L}.\frac{d\vec{L}}{dt} + \vec{L}\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

$$\Rightarrow 2\vec{L}.\frac{dL}{dt} = 2L\frac{dL}{dt}$$

Since $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$

$$\Rightarrow \frac{dL}{dt} = 0$$

$$\Rightarrow L = \text{contt.}$$

28. (b) $v = v_0 - \mu g t$

$$\omega = \omega_0 + \frac{2\mu g}{R} t$$

At pure rolling, $v = R\omega$

$$\Rightarrow \quad v_0 - \mu g t = R\left(\frac{v_0}{2R} + \frac{2\mu g}{R} t\right) \Rightarrow t = \frac{v_0}{6\mu g}$$

29. (b, c)

Let v and ω be the initial velocity of bullet and angular velocity of cylinder respectively. Applying conservation of angular momentum about the point of contact of cylinder with floor

$$mu.2R = mv.2R + \left(I_{CM} + MR^2\right)\omega$$

$v = 2R\omega$ and $I_{CM} = MR^2$

Substituting and solving, gives,

$$v = \frac{8mu}{8m + 3M}, \text{ and}$$

COMPREHENSION

30.

31.

32.

33.

34. Velocity of cylinder as it just comes out of water.

$v = \sqrt{2g\ell}$ by work energy theoram

hence height above the surface of liquid= *l*

total height = $\ell + \frac{\ell}{2} = \frac{3\ell}{2}$

35. Time for $\frac{3\ell}{4} \Rightarrow \frac{1}{4}\frac{2\pi}{2}\sqrt{\frac{\ell}{g}} = \frac{\pi}{4}\sqrt{\frac{\ell}{g}}$; time for $\frac{\ell}{4} \Rightarrow \frac{1}{2}\sqrt{\frac{\ell}{g}}\sin^{-1}\left(\frac{1}{3}\right)$

Hence total time $\Rightarrow \frac{\pi}{4}\sqrt{\frac{\ell}{g}} + \frac{1}{2}\sqrt{\frac{\ell}{g}}\sin^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow \sqrt{\frac{\ell}{g}}\left(\frac{\pi}{4} + \frac{1}{2}\sin^{-1}\left(\frac{1}{3}\right)\right)$

SUBJECTIVE TYPE QUESTIONS

36. Mg - T= Ma

T - mg = ma

T = $\frac{2Mmg}{(M+m)}$

$\Rightarrow \frac{T}{A} = \frac{2Mmg}{A(M+m)} = \frac{2mg}{A\left(1+\frac{m}{M}\right)} = 2x10^9$

$\Rightarrow M = 1.86Kg$

37. Relative motion between block and table will start when

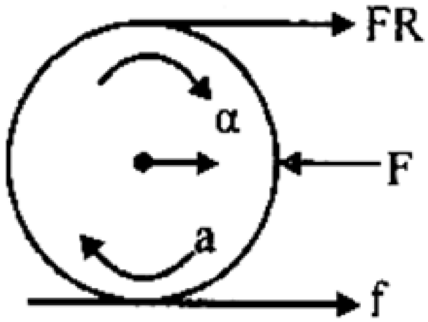
$m\omega^2 r\sin\theta = \mu(mg + m\omega^2 r\cos\theta)$ ..... (i)

$\omega = \alpha t$ ..... (ii)

solving (i) and (ii) t = $\sqrt{500}$ = 22.4 sec.

38. 2

$f = ma$, $FR - fr = mR^2 \frac{a}{R}$



$f = \frac{F}{2}$, $a = \frac{F}{2m}$

$f = 2N$