

FULL MECHANICS SOLUTION

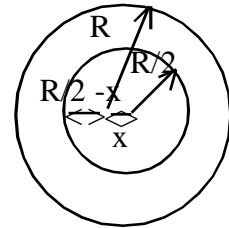
SINGLE CHOICE QUESTIONS

1. Minimum force to move the body = $\frac{\mu mg}{\sqrt{1+\mu^2}}$; $\Rightarrow \mu = \frac{3}{4}$

2. $\tau = mg \int v \cos \theta dt$; $\Rightarrow \frac{dL}{dt} = mgv \cos \theta$; $\Rightarrow L = \frac{mgv \cos \theta t^2}{2}$

3. $v^2 = \omega^2 (A^2 - x^2)$ & $a = -\omega^2 x \Rightarrow v^2 = -\frac{a}{x} (A^2 - x^2)$
 $\Rightarrow A = \sqrt{x^2 - \frac{xv^2}{a}}$

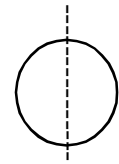
4. $\frac{4}{3}\pi R^3 \rho g - \frac{4}{3}\pi \frac{R^3}{8} \rho g = \frac{4}{3}\pi \left(\frac{R^3}{8} - \frac{R^3}{8} \right) \rho g$; $\Rightarrow \frac{4}{3}\pi R^3 \rho g - \frac{4}{3}\pi \frac{R^3}{8} \rho g = -\frac{x}{7} \Rightarrow x = \frac{7R}{16}$



5. Using perpendicular axis theorem,

$$2I_1 = M'R^2; \quad I_1 = M'R^2/2; \quad I_2 = \frac{M'R^2}{2} + M'R^2 = \frac{3}{2}M'R^2$$

$$I = \frac{I_2}{2} = \frac{3}{2} \frac{M'R^2}{2} = \frac{3}{2}MR^2$$



6. Upward force due to tension on 3 kg block is $F/2$ which should be greater than $3g$ and upward force on 2 kg block is $F/4$ which should be greater than $2g$

7. Avg. speed $3 = \frac{\sqrt{v_r^2 t^2 + v_{mr}^2 t^2}}{t}$

$$\Rightarrow v_r^2 + 5 = 9; \quad \Rightarrow v_r = 2 \text{ m/s}$$

8. $v_{\text{avg}} = \frac{\frac{1}{2}x \frac{t}{2} xv + \frac{1}{2}xv}{t} = \frac{3v}{4}$

9. $\frac{m}{2}g - T = \frac{m}{2}a \quad \dots (i)$

$T \cos 60^\circ = \frac{ma}{\cos 60^\circ}; \dots (ii)$ Solving (i) and (ii) accelerating o ring $\frac{2g}{9}$

10. Work done by all the forces on the block equal to change in kinetic energy.

11. No effect of 'a' and 'g' on time period of spring pendulum.

12. Work done by friction = $\int_0^x \vec{F} \cdot d\vec{s} = \int_0^x \mu mg \cos \theta \frac{dx}{\cos \theta}$
 $= \mu mgx = 20J$

13. Conservation of energy $\frac{1}{2}mv^2 - mg\frac{\ell}{2} = mg\frac{\ell}{2} + m\sqrt{3}\left[2\ell\frac{\sqrt{3}}{2}\right]g; \quad v = \sqrt{8g\ell}$

14. Conservation of momentum

$$M\sqrt{2gh} - m\sqrt{2gh} = MV_1 + mV_2 \quad (1)$$

$$-1 = \frac{V_1 - V_2}{2\sqrt{2gh}} \quad \dots (2)$$

$$V_2 = 3\sqrt{2gh}$$

$$h' = \frac{V_2^2}{2g} = 9h$$

15. FBD of 'B' and 'C'

$$\Rightarrow T - 2g = 2a \quad \dots (1)$$

$$\text{and } 3g - T = 3a \quad \dots (2)$$

$$T = \frac{12g}{5}$$

For A $\Rightarrow T = mg$

$$2 \times \frac{12g}{5} = mg \Rightarrow m = 4.8kg$$

16. $W = \int_0^{\pi/2} f \cdot R d\theta = \frac{\mu mg R \pi}{2} = 1 \text{ joule}$

17. Let us assume cylinder is no moving then

$$T + f_s = mg \sin \theta$$

$$T \cdot R - f_s R = 0$$

$$\Rightarrow f_s = \frac{mg\sqrt{3}}{4}$$

but $(f_s)_{\max} = \mu N = \mu mg \cos \theta$

$$= 0.4 \cdot mg \times \frac{1}{2} = \frac{mg}{5}$$

$\therefore (f_s) < (f_s)_{\max}$, our assumption is wrong. So, friction existing must be kinetic

$$f_k = \mu mg \cos \theta = 0.4 \times mg \times \frac{1}{2} = \frac{mg}{5}$$

18. For full square about an axis passing through 'O' = $\frac{M\ell^2}{6}$

by symmetry for remaining portion it must be $\frac{3}{4} \left(\frac{M\ell^2}{6} \right) = \frac{M\ell^2}{8}$ 29.B

19. Apply work-energy theorem

$$mgz - \frac{1}{2} K \left(\frac{mg}{K} + z \right)^2 - \frac{1}{2} K \left(\frac{mg}{K} \right)^2 + Fz = 0 ; \Rightarrow z = 2F/K$$

20. $a = 2 + |t - 2|$; for $t \leq 2$; $a = 2 - t + 2$; $a = 4 - t$

$$dv = (4 - t)dt ; v = 4t - t^2/2 \quad \text{at } t = 2, v = 6 \text{ m/s.}$$

for $t > 2$; $a = 2 + t - 2 = t$; $\int_6^v dv = \int_2^t t dv ; v - 6 = [t^2/2]_2^t$

$$v = \frac{t^2}{2} + 4 \quad \text{at } t = 4, v = 12 \text{ m/s.}$$

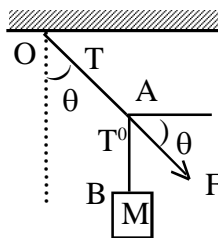
MULTIPLE CHOICE QUESTIONS

21. If at any instant elongation in spring is x and velocity of block is v from equilibrium position then

$$E_T \Rightarrow \frac{1}{2} kx^2 + \frac{1}{2} Mv^2 + \frac{1}{2} \int_0^L \frac{m}{L} dy \left(\frac{v}{L} y \right)^2$$

$$= \frac{1}{2} kx^2 + \frac{1}{2} Mv^2 + \frac{1}{2} \frac{m}{L} \frac{v^2}{L} \frac{L^3}{3}; \quad E_T \Rightarrow \frac{1}{2} kx^2 + \frac{1}{2} \left(M + \frac{m}{3} \right) v^2$$

22. $T \sin \theta = F \cos \theta$
 $T \cos \theta = F \sin \theta + T'$
 $T' - F \sin \theta = Mg$



$$23. \quad mg - \frac{2kq^2}{R^2} + T = \frac{mv^2}{R} \quad \dots (i)$$

$$-mgR + \frac{1}{2}mu^2 = mgR + \frac{1}{2}mv^2 \quad \dots (ii)$$

$$\text{if } T = 0 \Rightarrow u_{\min} = \sqrt{5gR - \frac{2Kq^2}{mR}}$$

$$24. \quad F = -\frac{dU}{dx} = -50(2x - 4)$$

$$\text{At mean position } F = 0 \Rightarrow x = 2\text{m}$$

$$U_{\min} = -20 \text{ J}; \quad a = -50 \times 2 (x-2)$$

$$\omega = 10 \text{ rad/sec}$$

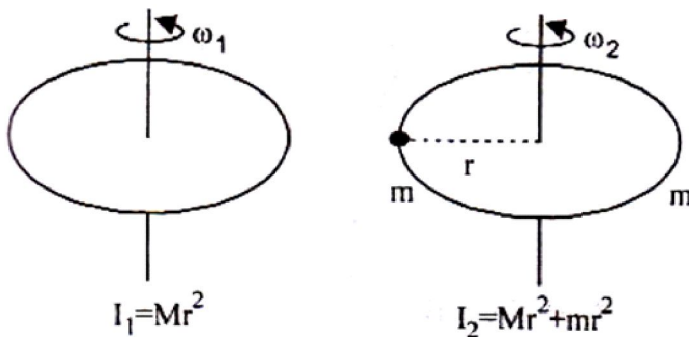
$$T = \pi/5 \text{ sec}$$

$$25. \quad \text{If the volume immersed initially is } (V/3). \text{ Then } \frac{V}{3}\rho g = mg \quad \dots (i)$$

If the volume immersed when the system accelerates is V' then

$$V'\rho \left(g + \frac{g}{2} \right) - mg = \frac{mg}{2} \Rightarrow V' = \frac{V}{3}$$

26. Since the objects are placed gently, therefore no external torque is acting on the system. Therefore angular momentum is constant.



$$\text{i.e., } I_1 \omega_1 = I_2 \omega_2$$

$$Mr^2 \times \omega_1 = (Mr^2 + 2mr^2) \omega_2$$

$$\therefore \omega_2 = \frac{M\omega}{M + 2m}$$

27. (a, b, c)

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Given that

$$\vec{\tau} = \vec{A} \times \vec{L} \Rightarrow \frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$$

From cross-product rule, $\frac{d\vec{L}}{dt}$ is always perpendicular to the plane containing

\vec{A} and \vec{L} .

By the dot product definition

$$\vec{L} \cdot \vec{L} = L^2$$

Differentiating with respect to time

$$\vec{L} \cdot \frac{d\vec{L}}{dt} + \vec{L} \cdot \frac{d\vec{L}}{dt} = 2L \frac{dL}{dt}$$

$$\Rightarrow 2\vec{L} \cdot \frac{d\vec{L}}{dt} = 2L \frac{dL}{dt}$$

Since $\frac{d\vec{L}}{dt}$ is perpendicular to \vec{L}

$$\Rightarrow \frac{dL}{dt} = 0$$

$$\Rightarrow L = \text{contt.}$$

28. (b) $v = v_0 - \mu g t$

$$\omega = \omega_0 + \frac{2\mu g}{R} t$$

At pure rolling, $v = R\omega$

$$\Rightarrow v_0 - \mu g t = R \left(\frac{v_0}{2R} + \frac{2\mu g}{R} t \right) \Rightarrow t = \frac{v_0}{6\mu g}$$

29. (b, c)

Let v and ω be the initial velocity of bullet and angular velocity of cylinder respectively. Applying conservation of angular momentum about the point of contact of cylinder with floor

$$mu \cdot 2R = mv \cdot 2R + (I_{CM} + MR^2) \omega$$

$$v = 2R\omega \quad \text{and} \quad I_{CM} = MR^2$$

Substituting and solving, gives,

$$v = \frac{8mu}{8m + 3M}, \text{ and}$$

COMPREHENSION

30.

31.

32.

33.

34. Velocity of cylinder as it just comes out of water.

$$v = \sqrt{2gl} \text{ by work energy theorem}$$

hence height above the surface of liquid = l

$$\text{total height} = l + \frac{l}{2} = \frac{3l}{2}$$

35. Time for $\frac{3l}{4} \Rightarrow \frac{1}{4} \frac{2\pi}{2} \sqrt{\frac{l}{g}} = \frac{\pi}{4} \sqrt{\frac{l}{g}}$; time for $\frac{l}{4} \Rightarrow \frac{1}{2} \sqrt{\frac{l}{g}} \sin^{-1} \frac{1}{3}$

$$\text{Hence total time} \Rightarrow \frac{\pi}{4} \sqrt{\frac{l}{g}} + \frac{1}{2} \sqrt{\frac{l}{g}} \sin^{-1} \frac{1}{3}$$

$$\Rightarrow \sqrt{\frac{l}{g}} \left[\frac{\pi}{4} + \frac{1}{2} \sin^{-1} \frac{1}{3} \right]$$

SUBJECTIVE TYPE QUESTIONS

36. $Mg - T = Ma$

$$T - mg = ma$$

$$T = \frac{2Mmg}{M+m}$$

$$\Rightarrow \frac{T}{A} = \frac{2Mmg}{A(M+m)} = \frac{2mg}{A \left(1 + \frac{m}{M} \right)} = 2 \times 10^9$$

$$\Rightarrow M = 1.86 \text{ Kg}$$

37. Relative motion between block and table will start when

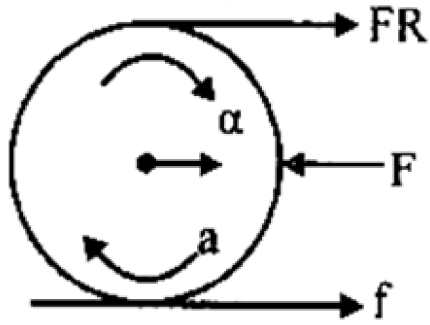
$$m\omega^2 r \sin \theta = \mu(mg + m\omega^2 r \cos \theta) \quad \dots (i)$$

$$\omega = \alpha t \quad \dots (ii)$$

$$\text{solving (i) and (ii)} \quad t = \sqrt{500} = 22.4 \text{ sec.}$$

38. 2

$$f = ma, FR - fr = mR^2 \frac{a}{R}$$



$$f = \frac{F}{2}, a = \frac{F}{2m}$$

$$f = 2N$$