WINDOW TO JEE MAIN

TROGONOMETRIC EQUATIONS

Q.1 [A]

$$\Rightarrow \sin\alpha + \sin\beta = -\frac{21}{65} \qquad \ldots\ldots..(1)$$

$$\Rightarrow \cos\alpha + \cos\beta = -\frac{27}{65} \qquad \ldots\ldots..(2)$$

Squaring and adding we get

$$\Rightarrow (\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 = \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$$

$$\Rightarrow \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = \frac{1170}{4225}$$

$$\Rightarrow 2 + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{36}{130}$$

$$\Rightarrow 2 + 2\cos(\alpha-\beta) = \frac{36}{130}$$

$$\Rightarrow 2[1 + \cos(\alpha-\beta)] = \frac{36}{130}$$

$$\Rightarrow 2\left[2\cos^2\left(\frac{\alpha-\beta}{2}\right)\right] = \frac{36}{130}$$

$$\Rightarrow \cos^2\frac{(\alpha-\beta)}{4} = \frac{9}{130}$$

$$\Rightarrow \cos\frac{\alpha-\beta}{2} = \frac{-3}{\sqrt{130}} \text{ as } \pi < \alpha-\beta < 3\pi$$

Q.2 [D]

$$u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$$

$$\Rightarrow u^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2\sqrt{a^2\cos^2\theta + b^2\sin^2\theta}\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$$

Let $t = a^2\cos^2\theta + b^2\sin^2\theta$

Then $u^2 = a^2 + b^2 + 2\sqrt{\left(a^2+b^2\right)t - t^2}$

$$\Rightarrow \frac{du^2}{d\theta} = \frac{1}{\sqrt{\left(a^2+b^2\right)t - t^2}}\left(a^2+b^2-2t\right)\frac{dt}{d\theta}$$

$$\Rightarrow \text{and } \frac{dt}{d\theta} = \left(b^2 - a^2\right)\sin 2\theta$$

$$\Rightarrow \therefore \frac{du}{d\theta} = \frac{\left(a^2+b^2-2t\right)}{\left(\sqrt{\left(a^2+b^2\right)t - t^2}\right)} \times \left(b^2 - c^2\right)\sin 2\theta$$

For maximum and minimum $\frac{du^2}{d\theta} = 0$

$\Rightarrow a^2 + b^2 = 2\left[a^2\cos^2\theta + b^2\sin^2\theta\right]$ and $\sin 2\theta = 0$

$\Rightarrow \cos 2\theta\left(b^2 - a^2\right) = 0$

$\Rightarrow \theta = 0,\ \cos 2\theta = 0$

Therefore u^2 will attain minimum at $\theta = 0$

and u^2 will attain maximum at $\theta = \frac{\pi}{4}$

$\therefore u^2_{min} = \left(a+b\right)^2$

$\Rightarrow u^2_{max} = 2\left(a^2+b^2\right)$

$\Rightarrow$ Now, $u^2_{max} - u^2_{min} = 2\left(a^2+b^2\right) - \left(a+b\right)^2 = \left(a-b\right)^2$

Q.3 [A]

$2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow \left(2\sin x - 1\right)\left(\sin x + 3\right) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ or $\sin x = -3$ not possible

Now, $\sin x = \frac{1}{2}$ will have

$\Rightarrow$ 2 solutions for $0 \le x < \pi$

$\Rightarrow$ 0 solutions for $\pi \le x < 2\pi$

$\Rightarrow$ 2 solutions for $2\pi < x < 3\pi$

Hence total 4 solutions.

Q.4 [C]

$\sin x + \cos = \frac{1}{2}$ ……… $0 < x < \pi$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \frac{\pi}{4} < x - \frac{\pi}{4} < \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} < x < \frac{3\pi}{4}$$

$\Rightarrow \therefore$ tan x lies in II quadrant

Now, $\sin x + \cos x = \frac{1}{2}$

Squaring both sides

$$\Rightarrow (\sin x + \cos x)^2 = \frac{1}{4}$$

$$\Rightarrow 1 + \sin 2x = \frac{1}{4}$$

$$\Rightarrow \sin 2x = \frac{-3}{4}$$

$$\Rightarrow \frac{2\tan x}{1+\tan^2 x} = -\frac{3}{4}$$

$$\Rightarrow 8\tan x = -3 - 3\tan^2 x$$

$$\Rightarrow 3\tan^2 x + 8\tan x + 3 = 0$$

$$\Rightarrow \tan x = \frac{-4 \pm \sqrt{7}}{3}$$

But tan x is in II quadrant

$$\Rightarrow \therefore \tan x = \frac{-4-\sqrt{7}}{3}$$

Q.5 [A]

$$\cos(\alpha+\beta) = \frac{4}{5} \Rightarrow \tan(\alpha+\beta) = \frac{3}{4}$$

$$\Rightarrow \sin(\alpha-\beta) = \frac{5}{13} \Rightarrow \tan(\alpha-\beta) = \frac{5}{12}$$

$$\Rightarrow \tan 2\alpha = \tan\left((\alpha+\beta)+(\alpha-\beta)\right)$$

$$\Rightarrow \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$\Rightarrow \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{36}{33}$$

Q.6 [D]

$$A = \sin^2 x + \cos^4 x$$

$$\Rightarrow \sin^2 x + \cos^2 x\left(1-\sin^2 x\right)$$

$$\Rightarrow 1 - \frac{1}{4}\sin^2 2x$$

Now $0 \le \sin^2 2x \le 1$

$\Rightarrow -1 \le -\sin^2 2x \le 0$

$\Rightarrow -\frac{1}{4} \le -\frac{1}{4}\sin^2 2x \le 0$

$\Rightarrow 1-\frac{1}{4} \le 1-\frac{1}{4}\sin^2 2x \le 1$

$\Rightarrow \frac{3}{4} \le 1-\frac{1}{4}\sin^2 2x \le 1$

$\Rightarrow \frac{3}{4} \le A \le 1.$

Q.7 [B]

$e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = k$

$\Rightarrow \therefore k = \frac{1}{k} - 4 = 0 \Rightarrow k^2 - 4k - 1 = 0$

$\Rightarrow k = 2 \pm \sqrt{5}$

$\Rightarrow k = 2 + \sqrt{5} + r$ not possible as it is greater than e.

$\Rightarrow \therefore k = 2 - \sqrt{5} = e^{\sin x} = -ve$

This is also not possible.

Hence no solution.